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M.Sc. I Sem. MATHEMATICS (PAPER–SECOND), 2017

(REAL ANALYSIS-I)

Time : Three Hours | Maximum Marks : 100

Note : Answer five questions in all. Short answer type Question No. 1 carrying 40 marks is compulsory. Answer one question carrying 15 marks from each Unit.

Note : The answer to short questions should not exceed 200 words and the answers to long questions should not exceed 500 words.

1. Answer the following : $4 \times 10 = 40$

(a) Define lower and upper Riemann stieltzes sums.

(b) Let $f : [a, b] \to R$ be defined by $f(x) = k$ and g a monotonically non-decreasing function on [a, b]. Prove that the integral

$\int_a^b f dg$ exists

and $\int_a^b f dg = k[g(b) - g(a)]$

(c) State Abel's test the convergence of the product of series $\Sigma u_n v_n$.

(d) Show that the series

$$\sum \frac{\sin nx}{n}$$

Converges for all real values of x

(e) Define pointwise convergence of a sequence.

(f) Lef {fn} be a sequence defined by

$fn : R \to R$ such that $fn(x) = \frac{x}{n}, \forall x \in R, n \in N$. Show that the sequence converges pointwise to the zero function.

(g) State Mn-test

(h) What do you understand by the differentiability of a function $F : A \to R^m$ where A is an open subsets of R^n.

(i) State Implicit function theorem.

(j) Define Jacobian of n functions $u_1, u_2, \dots u_n$ of n-variables $x_1, x_2, \dots x_n$.

Unit-I

2. Let f be a bounded function and g a be non decreasing function on [a, b]. Then prove that $f \in RS(\alpha)$ iff for every $\varepsilon > 0$, there exists a partitions p such

$$U(p,t,g) - L(p,f,g) < \varepsilon.$$

Or

3. Let $f_1, f_2 \in RS(g)$ on $[a, b]$. Show that $f_1 + f_2 \in RS(g)$ on $[a, b]$ and.

$$\int_a^b (f_1 + f_2)dg = \int_a^b f_1 dg + \int_a^b f_2 dg$$

Unit-II

4. State and prove the Dirichlet's test for the convergence of the product series $\Sigma u_n v_n$.

Or

5. If a series Σu_n is convergent, then show that the product series $\sum\left(\frac{n+1}{n}\right)u_n$ is also convergent.

Unit-III

6. Let g be a monotonically increasing function on [a, b] and let $\{f_n\}$ be a sequence of real valued functions defined on [a, b] such that $f_n \in RS(g)$ on [a, b] for n = 1, 2, ... If $f_n \to f$ uniformly on [a, b], then prove that $f \in RS(g)$ and $\int_a^b f dg = \lim_{n\to\infty} \int_a^b f_n dg$.

Or

7. State and prove Tauber's theorem.

Unit-IV

8. If x = r sinq.cosf. y = r sinq sinf z = r cosq, then find the Jacobian $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$

OR

9. Find the maximum and minimum radii vectors of the section of the surface

$(x^2 + y^2 + z^2) = a^2x^2 + b^2y^2 + c^2z^2$

By the plane $lx + my + nz = 0$