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M.Sc. I Sem. MATHEMATICS (PAPER–FOURTH), 2018

(COMPLEX ANALYSIS)

Time : Three Hours *Maximum Marks* : 100

Note : Answer **five** questions in all. Short answer type **Question No.1** carrying 40 marks is compulsory. Answer **one** question carrying 15 marks from each Unit. The answers to short questions should not exceed 200 words and the answers to long questions should not exceed 500 words.

1. Answer all the questions : 4×10=40

(a) If $\overline{f(\bar{z})} = f(z)$, Prove that f(x) is real.

(b) Let H be the upper half plane. Show that the map

$f : z \to \dfrac{z-i}{z+i}$ is an isomorphism of H with the unit disc.

(c) Define a natural boundary.

(d) Define analytic continuation along a curve.

(e) Show that the function defined by $f_1(z) = \int_0^{\infty} t^{3-zt} e \, dt$ is

analytic at all points z for which Re(z)>0. Find also a function which is the analytic continuation of $f_1(z)$ into the left hand plane R(z)<0 where R(z)means the real part of z.

(f) If f(z) is an entire function and $f(0) \neq 0$. Show that $f(z)=f(0)\, P(z) e^{P(z)}$ Where P(z) is a product of primary factors.

(g) Define integral function. If f(z) is an entire function which is never zero then show that f(z) must be of the form $e^{g(z)}$.

(h) Find the order of the function e^{az}, $a \neq 0$

(i) Define infinite product. Show that a necessary condition for the convergence of an infinite product.

$\pi(1+a_n)$is $\lim_{n\to\infty} a_n = 0$

(j) Show that an integral function affairs every finite value with at most one possible exception.

UNIT-I

2. If f(z) is analytic for |z|<| and satisfies the conditions 1 f(z)|≤|, f(0)=0, then |f(z)|≤|z| and | f'(0)|≤|. Equality sign holds only if f(z) is a linear transformation $w=e^{i\alpha}z$, where α is a real constant. Prove it. 15

OR

3. State and prove Schwartz's reflection Principal. 15

UNIT-II

4. State and Prove Riemann mapping theorem. 15

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OR

5. Let $\phi \subset \text{Hol}(U)$, and assume that ϕ is uniformely bounded on compact sets in U. Show that ϕ is relatively compact.

UNIT-III

6. Let U be bounded. Let $f : U \to D$ be an isomor-phism with the disc, and let α_1, α_2 be two distinct boundary points of U which are accessible. Suppose f is extended to α_1 and α_2 by continuity. Show that. 15

$$f(\alpha_1) \neq f(\alpha_2)$$

OR

7. Show that when b is real, the Series

$$\frac{1}{2}\log(1+b^2) + i\tan^{-1}b + \frac{2-ib}{1+ib} - \frac{1}{2}\left(\frac{z-ib}{1+ib}\right)^2 + \ldots$$

Is the analytic continuation of the function defined by the Series.

$$Z - \frac{Z^2}{2} + \frac{Z^3}{3} - \ldots\ldots.$$

UNIT-IV

8. Use mittagp-leffler's expansion theorem, prove that 15

$$\cot z = \frac{1}{z} + 2z\sum_{n=1}^{\infty} \frac{1}{z^2 - n^2\pi^2}$$

OR

[illegible] a_n real and $-1 < a_n \leq 0$. Show that the series $\sum a_n$ and the [illegible] converge or diverge together.