2013-16

Time : 4 hours

Full Marks : 80

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

Answer any **five** *questions.*

1. (a) Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction.

   (b) Obtain the principal conjunctive normal forms of

   $(-P \Rightarrow r) \wedge (q \Rightarrow p)$ without using truth table.

2. (a) What is principle of mathematical induction? Using this principle prove that

   $6^{n+2} + 7^{2n+1}$ is divisible by 43 for each positive integer.

(a) Negate the statement :

For a real number x, if $x > 3$ then $x^2 > 9$

(b) Prove that

(i) $A - B = A \cap B'$

(i) $(A \cup B)' = A' \cap B'$

(b) A computer company must hire 20 programmers to handle system programming jobs and 30 programmers for application programming of those hired. 5 are expected to perform jobs of both types. How many programmers must be hired?

4. (a) Define equivalance relation and show that if R and S are equivalence relation on the set A, $R \cap S$ is also an equivalence relation on A.

   (b) Use the Euclidean algorithm to find the greatest common divison of each pair of integers.

   (i) 12, 18

   (ii) 272, 1479

5. (a) Define inverse function of a function and composition of functions with examples.

   (b) Let $f : Z \to Z$ be a function defined by $f(x) = 2x + 3$, Let $g : Z \to Z$ be a function defined by $g(x) = 3x + 2$

   Find fog and gof.

6. Define primitive recursion.

   Show that function $f(x,y) = x + y$ is primitive recursion function. Hence compute the value of f (2, 4).

7. (a) Define abelian group. Show that the set {1, 2, 3, 4, 5} is not a group under addition and multiplication.

   (b) What is difference between semigroup and monoid? Give at least one examples each of them.

8. (a) Define boolean algebra.

   Show that $a + (a, b) = a$

   (b) Determine the conjunctive normal form of the Boolean function.

   $f(x, y, z) = (x, y' + x.z)' + z'$

9. (a) Define lattice with two examples.

   (b) In any lattice, show that

   (i) $a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c)$

   (ii) $a \vee (b \wedge c) \leq (a \vee b) \wedge (a \vee c)$.

10. Write short notes on the following :

   (i) Grammars and Languages

   (ii) Boolean Expression

   (iii) Euler graph

   (iv) Inference theory of the predicate calculus.

——— × ———