2012-15

Time : 4 hours

Full Marks : 80

Candidates are required to give their answers in their own words as far as practicable.

The questions are of equal value.

Answer any five questions.

1. (a) (i) Prove that :

$$\sim(p \wedge q) \rightarrow (\sim p \vee (\sim p \vee q)) = \sim p \vee q$$

(ii) Show that

$$(p \rightarrow q) \rightarrow \left[\left(p \vee (q \wedge r)\right) \leftrightarrow q \wedge (p \vee r)\right]$$

is a tautology.

(b) Check the validity of the following argument :

$$\begin{array}{l} p \\ q \\ p \wedge q \rightarrow r \vee q \\ \underline{p \wedge q \rightarrow r \wedge q} \\ r \end{array}$$

(c) Which of the following are statements ?

(i) 8 is greater than 3

(ii) Blood is green

(iii) It is raining.

(iv) What are you studing ?

(v) $2 < x < 6$

(vi) The sun will come out tomorrow.

(vii) $x^2 - 5x + 6 = 0$

(viii) Ramesh is poor but honest.

(b) Form the negation of each of the following :

(i) For all positive integers x, we have $x + 2 > 8$

(ii) All men are honest or some man is a thief.

(iii) There is at least one person who is happy all the times.

(iv) The sum of any two integers is an even integer.

3. (a) Prove by principle of mathematical induction that :

$2 + 4 + 6 + ... + 2n = n(n + 1)$

(b) Let R = {(4,5),(1,4),(4,6),(7,6),(3,7)}. find

(i) R^{-1} (ii) ROR

(iii) R^{-1}OR

4. (a) Show that the mapping :

$f : \mathrm{R} \rightarrow \mathrm{R}$: $f(x) = \cos x$

is neither one-one nor onto.

(b) A class has 175 students. The following is the description showing the number of students

studing one or more of the following subjects in this class.

Mathematics 100, Physics 70, Chemistry 46, Mathematics and Physics 30, Mathematics and Chemistry 28, Physics and Chemistry 23, Mathematics, Physics and Chemistry 18.

Find :

(i) How many student are enrolled in Mathematics alone. Physics alone and Chemistry alone.

(ii) The number of students who have not offered any of these three subjects.

5. (a) Define semigroup and monoid. Give example of each. If R be the set of real numbers and let * be an operation defined on R by :

$a * b = |a - b|$ for all $a, b \in R$

check whether (R,*) is a semigroup ?

(b) Construct a grammar for the language

$L = \{aaaa, aabb, bbaa, bbbb\}$

Also find L_1L_2 if $L_1 = \{a, b^2\}$ and $L_2 = \{a^2, ab, b^2\}$

6. (a) Show that the set $G = \{1, -1, i, -i\}$ forms a group with respect to multiplication of complex numbers as binary operation. Is this group an abelian group ? Find the order of the group.

(b) Find the language L(G) over $\{a, b\}$ generated by the grammar G = ({a,b},{S,C},S,P) where P consists of

$S \to aCa$, $C \to aCa$ and $C \to b$

(a) Let S = {1,2,3} and P(S) be its power set. Show that partially ordered set $(P(S), \subseteq)$ is a lattice represent the lattice by appropriate diagram.

(b) Show that the set B = {0,1} together with the operations +,* and ′ defined by the follwing tables is a Boolean algebra.

+	0	1
0	0	1
1	1	1

*	0	1
0	0	0
1	0	1

′	
0	1
1	0

8. (a) Simplfy the boolean function

$f = ab'cd + cb + cd' + ac' + a'bc' + b'c'd'$

(b) Explain in brief finite state machine by an example.

9. Define following :

(a) Simple Graph (b) Walk

(c) Path (d) Circuit

(e) Euler Graph (f) Hamiltonian graph

(g) Tree (h) Cutsets

10. Write short notes on the following :

(a) Recursion

(b) Polish expressions

(c) Partially ordered set

(d) Karnaugh map