C (Printed Pages 4)

Roll No. _____

21/1451

M.A./M.Sc. (First Semester)

Examination, 2021

MATHEMATICS

Fifth Paper

(Hydrodynamics)

समय : 1:30 घण्टे पूर्णांक : 100

Time : 1:30 Hours] *[Maximum Marks : 100*

नोट : **खण्ड-अ** एवं **खण्ड-ब** में दिये गये निर्देशों के अनुसार प्रश्नों के उत्तर दीजिए। लघु उत्तरीय प्रश्नों के उत्तर की अधिकतम् सीमा 150 शब्द एवं दीर्घ उत्तरीय प्रश्नों के उत्तर की अधिकतम सीमा 350 शब्द है।

Note : Attempt questions of **Section-A** and **Section-B** as directed. The answers to short questions should not exceed 150 words and the answers to long questions should not exceed 350 words each.

P.T.O.

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Section-A

Note : Attempt any **five** questions: 10×5=50

1. Answer any **five** questions.

(i) Define Stream lines and path lines. Also write the differential equation for a given streamline. 7+3

(ii) What is equation of continuity? Write the equation of continuity for homogeneous and incompressible fluid. 7+3

(iii) Give examples of rotational and irrotational flows. 5+5

(iv) Prove that stream function ψ is constant along stream lines. 10

(v) Define fixed and moving boundaries. 10

(vi) Define sources in two dimensions. Also define the strength of a given source. 5+5

(vii) Write the statement of circle theorem. 10

(viii) Write the statement of theorem of Blassins. 10

(ix) Derive complex potential ω for a doublet of strength μ. 10

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Section-B

Note : Answer any **two** questions: 25×2=50

2. Prove that $\frac{d\rho}{dt} + \rho \nabla.\bar{q} = 0$

where ρ and $\bar{q}$ has usual meaning. 25

3. Prove that the ellipsoid

$$\frac{x^2}{a^2 k^2 t^{2n}} + k t^n \left(\frac{y^2}{b^2} + \frac{z^2}{c^2}\right) = 1$$

is a possible form of the boundary surface of a liquid. 25

4. Derive Euler's equations of motion. 25

5. Prove that the complex potential w for a two dimensional source of strength m placed at origin is given by

$w = -m \log z$. 20+5

If a number of sources of strengths m_1, m_2, m_3 etc. are located at points a_1, a_2, a_3----, etc. then find the complex potential of motion.

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6. A region is bounded by a fixed quadrantal arc and its radii, with a source and an equal sink at the ends of one of these bounding radii. Show that the motion is given by

$$w = -m \log \frac{Z^2 - a^2}{Z}$$

and prove that the stream line leaving either the source or the sink at an angle α with the radius is $r^2 \sin(\alpha+\theta) = a^2 \sin(\alpha-\theta)$. 15+10

7. A circular cylinder of radius 'a' moving in an infinite mass of liquid at rest at infinity, with velocity U in the direction of x-axis. Prove that the complex potential for streaming motion past a fixed solid circular cylinder of radius 'a' is given by 25

$$w = U Z + \frac{a^2}{Z}$$

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