B. Sc. (Part-III) Examination, 2016

Mathematics- Fourth Paper (Optional)

(C) Differential Geometry and Tensor Analysis

Note :- Answer any five questions in all. Question No. 1 is compulsory. Answer one question from each unit. Marks allotted to each question are indicated in the right hand margin.

1. Answer the following in brief : $3.5 \times 10 = 35$
   (i) Define Osculatory plane at a point of a space curve.
   (ii) Prove that a necessary and suficient condition for a curve to be a straight line is that curvature at each of its point vanishes.
   (iii) Define involute and evolute of a space curve.
   (iv) Define geodesic curvature of a curve on a surface.
   (v) Define umbilic point.
   (vi) Define inner product of two tensors. Show that inner product of a contravariant vector and a covariant vector is a scalar quantity.
   (vii) Show that $S' = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$ is a differential manifold.
   (viii) Show that a tensor of type (0,2) can be written as a sum of a symmetric tensor and a skew symmetric tensor.
   (ix) Prove that the metric tensor g_{ij} is Covariant constant.
   (x) Define Christoffel symbol of first kind. Show that these quantities are not components of a tensor.

Unit-I

2. (a) Calculate curvature and torsion of the space curve γ given by 5

$$\gamma(t) = (t, t^2, t^3).$$

at a point 't'.

   (b) Show that a necessary and sufficient condition for a space curve to be a helix is that its curvature and torsion are in constant ratio at each of its points. 5 **Or**

3. (a) Show that the tangent to the locus of the centre of the osculating sphere passes through the centre of the osculating circle.
   (b) State and prove Serret-Frenet's formula for a space curve parametrized by arc length.

Unit-II

4. (a) Define a regular surface. Show that graph of a smooth function $f : \mathbb{R}^2 \to \mathbb{R}$ is a regular surface. 5
   (b) Calculate the fundamental magnitudes of the right helicoid given by- 5

$$\gamma(u,\phi) = (u\cos\phi, u\sin\phi, c\phi).$$

Or

5. (a) Show that the condition that the directions given by $P_{\alpha\beta} du^{\alpha} du^{\beta} = 0$ are orthogonal is 5

$$g^{\alpha\beta} P_{\alpha\beta} = 0.$$

(b) State and prove Meunser's theorem. 5

Unit-III

6. State and prove Gauss-Bonnet theorem. 10 **Or**
7. State and prove the equation of Gauss and the equation of Weingarten. Also writes few words on the importance of these equations.

Unit-IV

8. (a) State and prove Quotient Law. 5
   (b) Define geodesics of a regular surface. Find all geodesics of a right circular cylinder. 5 **Or**
9. (a) Prove that covariant derivative of a contravariant vector is a mixed tensor. 5+5
   (b) Show that the law of transformation of a mixed tensor of type (1,1) possesses the group property.