B. Sc. (Part-III) Examination, 2016
Mathematics- Second Paper
(Second Analysis)

Note :- Answer five questions in all. Question No. 1 is compulsory. Answer one question from each unit. Marks allotted to each question are indicated in the right hand margin.

1. Answer the following : $3.5 \times 10 = 35$

(i) Prove that whenever the limit of one valued function exists at a point it is unique.

(ii) Find the radius of convergence of the power series

$$\sum z^n / n^n$$

(iii) Find the harmonic conjugate of the harmonic function

$$u = \frac{1}{2}\log(x^2 + y^2)$$

(iv) Find the bilinear transformation which transform the point $z = 2, 1, 0$ into $w = 1, 0, i$.

(v) Explain the concept of isogonal and conformal mapping.

(vi) Evaluate $\int_C dz$ (ab-initio)

(vii) Expand the series of the function

$$f(z) = \frac{1}{z^2 - 3z + 2}$$

in the region $1 < |z| < 2$

(viii) Find the residue of function $\frac{z^2}{z^2 + a^2}$ at $z = ia$

(ix) Define natural boundary.

(x) Distinguish between pole and essential singularity.

Unit-I

2. (a) Prove that the function $u+iv = f(z)$

where $f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}$,

$z \neq 0, f(0) = 0$

is continuous and Cauchy-Riemann equations are satisfied at the origin; yet $f'(0)$ does not exist. 6

(b) off $f(z) = u+iv$ is analytic function in domain D, prove that the curve $u = $ const. $v = $ const. forms two orthogonal families. 4 **Or**

3. (a) Prove that the power series represents an analytic function inside its circle of convergence. 6

(b) Obtain Cauchy-Riemann equations for an analytic function. 4

Unit-II

4. (a) To prove that at each point z of a domain where f(z) is analytic and $f'(z) \neq 0$, the mappig $w = f(z)$ is conformal. 5

(b) Show that the relation $w = \frac{5-4z}{4z-2}$ transform the circle $|z| = 1$, into a circle of radius unity in w-plane. 5 Or

5. (a) Find all mobius transformations which transform the unit circle $|z| \leq 1$ into the unit circle $|w| \leq 1$. 5

(b) Show that by means of the transformation 5

$$w = \left(\frac{z-ic}{z+ic}\right)^2 \text{ (c real)}$$

the upper half of w-plane may be made to correspond to the interior of a certin semi-circle in the z-plane.

6. (a) Off a function f (z) is analytic inside and on a simple closed contour c, then prove that $\int_c f(z)\,dz = 0$ 5

(b) Off a function f(z) is analytic for finite values of z and is bounded, then prove that f(z) is constant. 5 **Or**

7. (a) State and prove Maximum Modulus theorem. 5

(b) State and prove Cauchy's integral formula for the derivative of an analytic function. 5

Unit-IV

8. (a) What kind of singularity do the following functions have :

(i) $\frac{1}{\sin z - \cos z}$ at $z = \frac{\pi}{4}$

(ii) $\sin\frac{1}{z}$ at $z = 0$

(b) Prove that : 6 **Or**

$$\int_0^\infty \frac{\cos mx}{1+x^2}dx = \frac{\pi e^{-m}}{2}, m > 0$$

9. (a) Off $0 < b < 1$, show that the series : 5

$$\frac{1}{2}\log(1+b^2) + i\tan^{-1}b + \frac{z-ib}{1+ib} - \frac{1}{2}\left(\frac{z-ib}{1+ib}\right)^2 + \ldots\ldots\ldots$$

is analytic continuation of the function defined by series,

$$z - \frac{1}{2}z^2 + \frac{1}{3}z^3 - \frac{1}{4}z^4 + \ldots\ldots\ldots$$

(b) Prove that 5

$$\int_0^\infty \frac{\sin x}{x}\, dx = \frac{\pi}{2}$$