B. Sc. (Part-III) Examination, 2017
Mathematics- Second Paper
(Complex Analysis)

Note :- Answer five questions in all. Question No. 1 is compulsory. Answer one question from each unit. Marks allotted to each question are indicated in the right hand margin.

1. Answer the following : 3.5×10

(i) Find the points where the complex valued function $f(z) = xy^2 + ix^2y$ is analytic.

(ii) Find the image of the circle $|z - 3i| = 3$ under the transformation

$$w = \frac{1}{z}.$$

(iii) Is the transformation w = conformal at the origin?

(iv) Find the fixed points of $w = \frac{3z-4}{z-1}$.

(v) Evaluate f (2) and f (3) where

$$f(a) = \int_C \frac{2z^2 - z - 2}{z - a}\, dz$$

and C is the circle $|z| = 2.5$.

(vi) Evaluate

$$\int_C \frac{e^{2z}}{(z+a)^4}$$

where C is the circle $|z| = 2$.

(vii) Show that the series

$z(1-z)+z^2(1-z)+z^3(1-z)+ \cdots \infty$

converges for $|z| < 1$. Determine whether it converges absolutely or not.

(viii) Find the Taylor's series expansion of $\left(\frac{1}{z-2} - \frac{1}{z-1}\right)$ in the region $|z| = 1$.

(ix) Show that the point $z = 0$ is an essential singularity of the function $f(z) = ze^{1/z^2}$.

(x) If f(z) has a pole of order three at z = a, Write down the formula for Res [f(a)].

Unit-I

2. (a) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin even though the Cauchy-Riemann equations are satisfied thereof. **5**

(b) Prove that $u = x^2 - y^2$ and $v = \frac{y}{x^2 + y^2}$ are harmonic functions of (x,y) but are not harmonic conjugates. **5 Or**

3. (a) Determine the analytic function whose real part is e^{2x} (x cos 2y-y sin 2y). **5**

(b) Show that under the transformation $w = \frac{z-i}{z+i}$, real axis in the z-plane is mapped into the circlewl = 1. Which portion of the z-plane corresponds to the interior of this circle. **5**

Unit-II

4. (a) Find the value of $\int_0^{1+i} (x-y+ix^2)\,dz$ along : **5**

(i) The staight line from z = 0 to z+i

(ii) The real axis from z = 0 z = 1 and then along a line parallel to the imaginary axis from z = 1 to z = 1+i.

(b) If f(Z) is analytic in the region D between two simple closed curves C and C_1, prove that $\int_C f(z)\,dz = \int_{C_1} f(z)\,dz$. **5 Or**

5. (a) State and prove Cauchy integral formula. **5**

(b) Evaluate the following by using Cauchy integral formula

$\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)}\,dz$ where C represents the circle $|z| = 3$. **5**

Unit-III

6. (a) Find the Laurent expansion of $f(z) = \frac{z}{(z+1)(z+2)}$ about the singularity z = -2, specify the region of convergence. **5**

(b) Use Rouche's theorem to show that the equation $z^5 + 15z + 1 = 0$ has one root in the disc $|z| < \frac{3}{2}$ and four roots in the annulus $\frac{2}{3} < |z| < 2$ **5 Or**

7. (a) Find the nature and location of the singularities of the function

$$f(z) = \frac{1}{z(e^z - 1)}$$

Prove that f (z) can be expanded in the form $\frac{1}{z^2}-\frac{1}{2z}+a_0+a_2 z^2$ $+a_4 z^4+\cdots$ where $0<|z|<2\pi$ and find the values of a_0 and a_2. 5

(b) Show that the series : 5

(i) $\sum_{n=0}^{\infty}\frac{z^n}{2^{n+1}}$, (ii) $\sum_{n=1}^{\infty}\frac{(z-i)^n}{(2-i)^{n-1}}$

are analytic continuations of each other.

Unit-IV

8. (a) State and prove the Cauchy's residue theorem. 5

(b) Use residue theorem to evaluate

$$\int_C \frac{z^2}{(z-1)^2(z+2)}\,dz$$

where C is the circle $|z| = 2.5$. 5 Or

9. (a) Distinguish the difference between isolated singularity, removable singularity and essential singularity of complex valued functions. 5

(b) Evaluate $\int_0^{2\pi}\frac{\cos 2\theta}{5+4\cos\theta}\,d\theta$ by the method of contour integration. 5