B. Sc. (Part-III) Examination, 2017
Mathematics- Fourth Paper (Optional)
(C) Differential Geometry and Tensor Analysis

Note :- **Answer any five questions in all. Question No. 1 is compulsory. Answer one question from each unit. Marks allotted to each question are indicated in the right hand margin.**

1. **Answer the following in brief :** $3.5 \times 10 = 35$
   (i) Find the equation of the osculating plane at a general point on the curve given by $r = (u, u^2, u^3)$.
   (ii) Show that Seret-Frenet formulae can be written in the form $t' = wt$, $n' = w \times n$, $b' = w \times b$ and determine w.
   (iii) Find the radii of curvature and torsion of the helix $x = a \cos u$, $y = a \sin u$, $z = a u \tan \alpha$.
   (iv) Find the equation for the tangent plane to the surface $z = x^2+y^2$ at the point (1, -1, 2).
   (v) For the paraboloid $r = (u, v, u^2-v^2)$ find the metric.
   (vi) Show that the curve bisecting the angles between the parametric curves are given by $E\, dv^2 - G\, dv^2 = 0$.
   (vii) Show that the Gaussian curvature of the surface given by $z = f(x,y)$ is
   $$\frac{rt - s^2}{(1+p^2+q^2)^2}$$
   (viii) Find the envelope of the plane $lx+my+nz = 0$ where $al^2+bm^2+cn^2 = 0$.
   (ix) Prove that the velocity of a fluid at any point is a contravariant tensor of rank one.
   (x) Show the $g_{ij}\, dx^i\, dx^j$ is an invariant.

Unit-I

2. (a) To show that the necessary and sufficient condition for the curve to be plane is $[\dot{r}, \ddot{r}, \dddot{r}] = 0$. 10
   (b) Find the Osculating plane, Curvature and Torsion at any point of the curve $x = a \cos 2u$, $y = a \sin 2u$, $z = 2a \sin u$. **Or**
3. (a) Show that a necessary and sufficient condition for a curve to be helix is that the ratio of the curvature and torsion is constant. **10**
   (b) If a curve lies on a sphere, show that ρ and σ are related by $\frac{d}{ds}(\sigma\rho') + \frac{\rho}{\sigma} = 0$.

Unit-II

4. (a) If the radius of spherical curvature of a curve is constant, Prove

that the curve either lies on the surface of a sphere of else has constant curvature. **10**

(b) Show that the involutes of a circular helix are plane curve. **Or**

5. (a) To show that a proper parametric transformation either leaves every normal unchanged or reverses every normal. **10**

(b) The metric remains invariant if the parameters u,v are transformed to the parameters u', v' by the relations of the forms $u' = \phi(u,v)$ $v' = \psi(u,v)$.

Unit-III

6. (a) Show that if there is a surface of minimum area passing through a closed space curve, it is necessarily a minimal surface. **10**

(b) To show that F = 0, M = 0 is the necessary and sufficient condition for the lines of curvature to be parametric curves. **Or**

7. (a) To find surface of revolution having constant negative Gaussian curvature. **10**

(b) Show that the principal radii of curvature of the surface $y \cos(z/a) = x \sin(z/a)$ are equal to $\pm (x^2+y^2+a^2)/a$. Find lines of curvature.

Unit-IV

8. (a) Prove that $\dfrac{\partial A_r}{\partial x^s}$ is not a tensor even though A_r is a covariant tensor of rank one. **10**

(b) Prove that the outer product of tensors is commutative and associative. **Or**

9. (a) A symmetric tensor of rank two has at most $\dfrac{1}{2}N(N+1)$ different components in V_N. **10**

(b) To show that Christoffel's symbol is not a tensor quantity.